25. On the Condition That the Roots of a Cubic Are Real or Imaginary

Author(s): T. G. Vyvyan
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$\therefore$ present value of an annuity of A for n years
$=$present value of a perpetual annuity of A − present value of a perpetual annuity of A to commence in n years

$$=\frac{A}{r}-\text{present value of } \frac{A}{r} \text{ due in } n \text{ years}$$

$$=\frac{A}{r}-\frac{A}{r}\times\frac{1}{R^n}=\frac{A(1-R^{-n})}{r}.$$

PROF. LONEY.

25. *On the condition that the roots of a cubic are real or imaginary.*

Let one real root of the equation

$$x^3+qx+r=0$$

be 2α; then the other roots may be written in the form $-\alpha+\beta$, $-\alpha-\beta$, where β may be real or a pure imaginary.

Now $$q=-4\alpha^2+\alpha^2-\beta^2=-3\alpha^2-\beta^2,$$
and $$r=2\alpha(\alpha^2-\beta^2),$$
$$\therefore\quad 4q^3+27r^2=-4\beta^2(9\alpha^2-\beta^2)^2.$$

Now $(9\alpha^2-\beta^2)^2$ is positive whether β is real or imaginary, and β^2 is positive or negative as the roots are real or imaginary.

Hence if $4q^3+27r^2$ is positive, the roots are imaginary; if zero, equal; if negative, real.

T. G. VYVYAN.

26. *On Continued Fractions.*

Consider the continued fraction

$$a_1+\frac{1}{a_2+}\,\frac{1}{a_3+}\ldots+\frac{1}{a_n}.$$

It is clear that, if p_n/q_n be the n^{th} convergent, both p_n and q_n are of the first degree in a_n; it is also clear that no reduction to lower terms can take place in the course of the work.

Hence $$\frac{p_n}{q_n}=\frac{Aa_n+B}{Ca_n+D}.\ \ldots\ldots\ldots\ldots\ldots\text{(i.)}$$

Now if a_n be made infinite, the convergent reduces to p_{n-1}/q_{n-1}.

Hence $$\frac{p_{n-1}}{q_{n-1}}=\frac{A}{C},$$
and $$A=p_{n-1},\quad C=q_{n-1}.\ \ldots\ldots\ldots\ldots\ldots\text{(ii.)}$$

Again, if a_n be made zero, the convergent reduces to p_{n-2}/q_{n-2}.

Hence $$\frac{p_{n-2}}{q_{n-2}}=\frac{B}{D},$$
and $$B=p_{n-2},\quad D=q_{n-2};\ \ldots\ldots\ldots\ldots\ldots\text{(iii.)}$$
therefore $$p_n=a_np_{n-1}+p_{n-2},$$
$$q_n=a_nq_{n-1}+q_{n-2}.$$

PROF. STEGGALL.

27. *If* $\frac{p_n}{q_n}$ *be the* n^{th} *convergent of the continued fraction* $\frac{a_1}{b_1+}\,\frac{a_2}{b_2+}\,\frac{a_3}{b_3+}\ldots$ *it may always be assumed that* $p_0=0$, $q_0=1$.

In using the formulae $p_n=b_np_{n-1}+a_np_{n-2}$, $q_n=b_nq_{n-1}+a_nq_{n-2}$ for obtaining the successive convergents, it is always to be understood that if any convergent $\frac{p_r}{q_r}$ is not in its lowest terms, it must not be reduced. The same condition applies to $\frac{a_r}{b_r}$. For the first two convergents we have

$$p_1=a_1,\quad q_1=b_1,\quad p_2=a_1b_2,\quad q_2=b_1b_2+a_2.$$